Model of fatigue damage in strain-rate-sensitive composite materials

by

Sanboh Lee
Department of Materials Science and Engineering
National Tsing Hua University
Hsin-Chu, Taiwan, Republic of China

Tinh Nguyen
Building and Fire Research Laboratory
National Institute of Standards and Technology
Gaithersburg, MD 20899 USA

and

Tze-jer Chuang
Ceramics Division
National Institute of Standards and Technology
Gaithersburg, MD 20899 USA

Reprinted from Journal of Materials Research, Vol. 18, No. 1, pp. 77-80, January 2003.

National Institute of Standards and Technology
Technology Administration, U.S. Department of Commerce

Model of fatigue damage in strain-rate-sensitive composite materials

Sanboh Lee[a)]
Department of Materials Science and Engineering, National Tsing Hua University, Hsin-Chu, Taiwan, Republic of China

Tinh Nguyen
Materials and Construction Research Division, National Institute of Standards and Technology, Gaithersburg, Maryland 20899

Tze-jer Chuang
Ceramics Division, National Institute of Standards and Technology, Gaithersburg, Maryland 20899

(Received 7 June 2002; accepted 10 October 2002)

A fatigue damage accumulation model based on the Paris law is proposed for strain-rate-sensitive polymer composite materials. A pre-exponent factor c_2/f and strain-rate-sensitive exponent n are introduced. Numerical analysis of the model was performed using experimental data obtained in the literature. Both factors were found to enhance fatigue damage accumulation. The analysis also revealed that the extent of damage increases with decreasing frequency and that the damage rate is more sensitive to the applied maximum stress than to the stiffness of the material.

I. INTRODUCTION

Advanced polymeric composites have increasingly gained applications as structural materials. Development of advanced methodologies for assessing the safety and reliability of these complex products has become urgent. Fatigue reliability under severe environments and extreme load is one of the most challenging issues facing the use of these materials for structural applications. Fatigue damage mechanisms in polymer composites have been studied extensively.[1–10] The evolution of damage in composite materials consists of matrix cracking, interface debonding, delamination, splitting, and fiber breaking. Until now, the effect of various parameters on the stage of damage process has not been thoroughly investigated. For example, Ye divided the damage process into two stages.[8] In the first stage, damage initially increases rapidly and then gradually slows down to reach a plateau. In the second stage, the damage increases rapidly until fracture. The duration of the second stage is very short, normally occupying about 20% of the total lifetime of the composite materials. Talreja also considered the damage degradation as a two-stage process separated by the characteristic damage state in which a stable crack pattern develops.[2] However, he suggested that the first stage is generally 80% of the total lifetime of the composites.

Because the fatigue behavior of advanced polymeric composites involves a very complicated process, it is very difficult, if not impossible, to thoroughly characterize the fatigue failure mechanisms of these materials simply based on stress analysis alone. However, several studies have used the Paris law, which can describe the fatigue of metallic alloys very well, to analyze fatigue behavior of polymer composites. For example, Wnuk[11] and Lowe *et al.*[12] have developed a Paris-law-based expression that accounts for both mode I critical stress intensity factor and viscoelastic response. Ogin *et al.* have applied the Paris law to analyze matrix cracking and stiffness reduction during the fatigue of (0/90) glass-fiber-reinforced plastic laminates.[7] They found that stiffness reduction is directly proportional to the density of cracks. Using the same concept, Ye has derived an expression for fatigue damage of short-fiber reinforced polymer composites.[8] The main objective of this paper is to develop a mathematical fatigue model for predicting the residual properties of strain-rate-sensitive composite materials under an action of cyclic load and a given environmental history.

II. DAMAGE ACCUMULATION MODEL

Due to the complexity of fatigue mechanisms, it is very difficult to characterize micro-damage in polymer composites in terms of deformation and stress state. To tackle this problem, it is customary to introduce a damage variable to characterize the fatigue damage state. The

[a)]e-mail: sblee@mse.nthu.edu.tw

damage variable D is generally described as a function of the maximum applied stress σ_{max}, number of loading cycles N, load ratio R, environmental conditions such as temperature T, and material properties such as stiffness E, as given below:

$$D = D(\sigma_{max}, N, E, R, T, \ldots) \quad . \tag{1}$$

Numerous approaches, including the use of residual strength[13] and remanent life,[14] have been introduced to describe the evolution of fatigue damage in composite materials. Change in material stiffness, which has been adopted by many researchers,[1,4,5] is utilized in this study to express fatigue damage in composite materials. The damage variable D is defined as

$$D = 1 - E/E_0 \quad . \tag{2}$$

Here, E_0 and E are the initial and current stiffness values of the material, respectively. Obviously, $D = 0$ means the material has no damage, and D is less than unity because E is always greater than zero before the material fails. $D = D_c$ (<1) represents a damage state at which catastrophic failure occurs.

Based on the Paris law, Wnuk[11,12] derived an expression for fatigue crack growth da/dN in polymeric materials that included both mode I critical stress intensity factor, K_{IC}, and viscoelastic response, i.e.,

$$\begin{aligned}\frac{da}{dN} &= \lambda_1\left(\frac{\Delta K}{K_{IC}}\right)^m + \frac{\lambda_2}{f}\left(\frac{\Delta K}{K_{IC}}\right)^n \\ &= \lambda_1\left(\frac{2\pi a S_a^2}{K_{IC}}\right)^m + \frac{\lambda_2}{f}\left(\frac{2\pi a S_a^2}{K_{IC}}\right)^n \quad ,\end{aligned} \tag{3}$$

where pre-exponent factors λ_1 and λ_2 and exponents m and n are the material constants, f is the test frequency, a is the crack length, N is the number of loading cycles, and S_a is the fatigue stress amplitude. The mode I critical stress intensity factor K_{IC} is a material constant, which is determined by the sample geometry, materials microstructure, and tensile strength.

Although the mechanical properties of a polymer are different from those of its counterpart composite material, the fatigue a-N curves for the two materials are similar to one another. For these materials, crack generation followed by crack propagation is the dominant damage mode. Crack size is, therefore, generally assumed to be proportional to the damage variable D. For polymer composites, the size and number of cracks increases with the number of loading cycles before catastrophic failure occurs. The stress intensity factor at the ith crack tip is

$$K_i = K_{iA} + \sum_{j \neq i} K_{ij} \quad , \tag{4}$$

where K_{iA} is due to applied load and K_{ij}, the stress intensity factor at the ith crack tip arises from other cracks j, debonding, delamination, etc. K_i has been found to increase with increasing square of maximum applied stress and decreasing distance between cracks.[7] The average distance between cracks is equal to the reciprocal of crack density, which is proportional to the damage variable. That is, K_i is proportional to σ_{max}^2/D. Following the above argument, we can define a damage accumulation function for polymer composite materials as:

$$\frac{dD}{dN} = c_1\left(\frac{\sigma_{max}^2}{D}\right)^m + \frac{c_2}{f}\left(\frac{\sigma_{max}^2}{D}\right)^n \quad , \tag{5}$$

where pre-exponent factors c_1 and c_2 and exponents m and n are material constants, σ_{max} is the maximum applied stress of each loading cycle, and c_2 and n correspond to the strain-rate-sensitive parameters. When $c_2 = 0$, Eq. (5) is reduced to that proposed by Ogin *et al.*[7] and Ye.[8] It should be noted that this equation is independent of R, the load ratio. Generally speaking, the parameter R plays an important role in metal fatigue behavior. However, in the case of polymeric composites, very few data are present to support the effect of R on the damage growth rate. Hence it seems that it is acceptable to exclude R in Eq. (5), at least in the case of polymeric composites we studied.

Assuming m > n and integrating Eq. (5) with the initial condition $D(N = 0) = 0$, we obtain

$$\begin{aligned}N &= \frac{\sigma_{max}^2}{(m+1)c_1}\left(\frac{D}{\sigma_{max}^2}\right)^{m+1} \\ &\quad + \sum_{i=1}^{\infty}\frac{\sigma_{max}^2}{[(m-n)i + m + 1]c_1}\left(-\frac{c_2}{c_1 f}\right)^i\left(\frac{D}{\sigma_{max}^2}\right)^{(m-n)i+m+1} \quad .\end{aligned} \tag{6}$$

According to Ye,[8] if the critical damage level D_c can be determined by a certain criterion, the duration of the first stage N_f can be predicted that covers most of fatigue life. On the other hand, if n > m, the solution of Eq. (5) with the initial condition $D(N = 0) = 0$ is

$$\begin{aligned}N &= \frac{\sigma_{max}^2}{(n+1)c_2}\left(\frac{D}{\sigma_{max}^2}\right)^{n+1} \\ &\quad + \sum_{i=1}^{\infty}\frac{\sigma_{max}^2}{[(n-m)i + n + 1]c_2}\left(-\frac{c_1}{c_2 f}\right)^i\left(\frac{D}{\sigma_{max}^2}\right)^{(n-m)i+m+1} \quad .\end{aligned} \tag{7}$$

Under this situation, the fatigue damage behavior is dominated by the strain-rate-sensitive parameters.

III. NUMERICAL RESULTS

For the random short-fiber sheet moulding compound (SMC) composite subjected to tensile fatigue loading, Ye[8] obtained a value of $c_1 = 1.21 \times 10^{-39}$ and m $= 7.43$

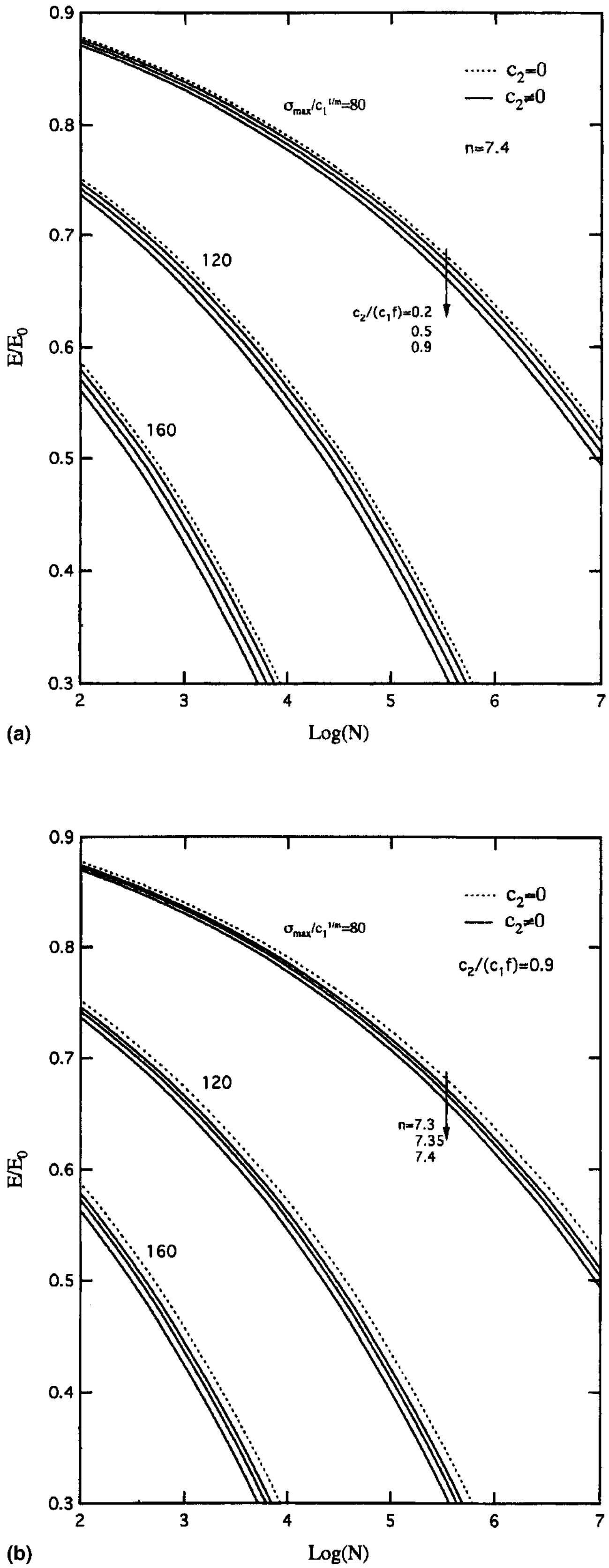

FIG. 1. Change of stiffness as a function of number of loading cycles: (a) frequency effect and (b) strain-rate-sensitive exponent effect.

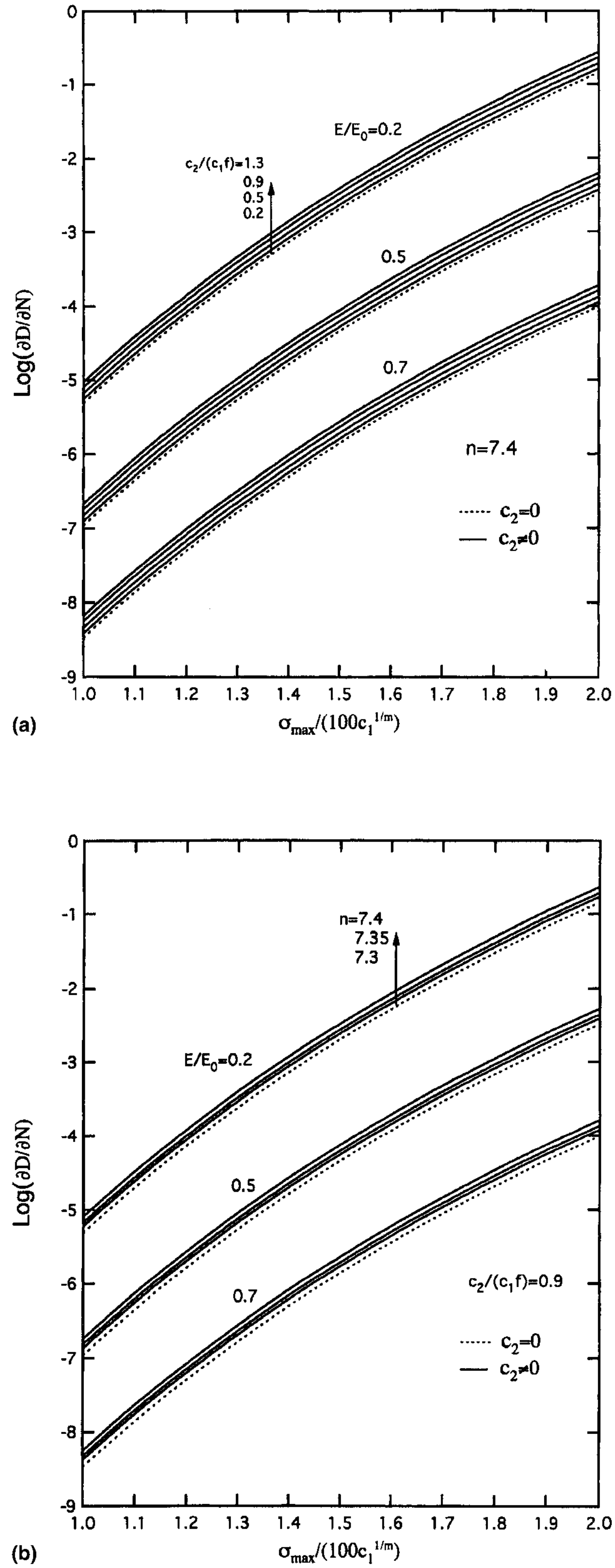

FIG. 2. Damage rate as a function of the maximum applied stress: (a) frequency effect and (b) strain-rate-sensitive exponent effect.

when $c_2 = 0$. In this paper, we use the data of Ye and investigate the influence of c_2/f and n in the second term of Eq. (5) on the fatigue life of polymer composites. Figure 1(a) shows the effect of c_2/f on the damage evolution where n = 7.4. For a given N and σ_{max}, the stiffness decreases with increasing c_2/f. If c_2 is constant, the stiffness increases with increasing frequency. Since the extent of damage increases with decreasing stiffness, these results imply that the extent of damage increases with decreasing frequency. It is reasonable to stipulate that the material has more time to respond to the viscoelastic deformation at the low frequency than at the high frequency. According to Fig. 1(a), the stiffness decreases with an increase in the number of loading cycles for a given maximum applied stress. Further, for a given number of loading cycles, the stiffness decreases with increasing maximum stress. If $D_c = 0.3$ for the critical damage variable (i.e., $E_c/E_0 = 0.7$), the number of loading cycles increases with decreasing maximum stress and increasing frequency. The effect of n on the damage evolution for $c_2/(c_1 f) = 0.9$ is displayed in Fig. 1(b). For a given number of loading cycles, the stiffness decreases with increasing n. As illustrated in Fig. 1, the second term of Eq. (5) always enhances the fatigue damage in composite materials.

In Figs. 2(a) and 2(b), the variations c_2/f and n on the curves of damage accumulation rate versus σ_{max} are presented. For a given σ_{max} and E, the damage accumulation rate increases with c_2/f. That is, the damage accumulation rate decreases with increasing frequency. It also decreases with increasing stiffness, but increases with increasing n and maximum applied stress.

IV. SUMMARY AND CONCLUSIONS

We developed a model based on the Paris law to investigate the fatigue damage accumulation for strain-rate-sensitive polymer composite materials. The analysis has shown that the fatigue damage of these materials is sensitive to the frequency and strain-rate-sensitive parameters. The damage rate is found to increase with a decrease in frequency. For the same maximum applied stress, the effect of strain-rate-sensitive parameters on the number of loading cycles is more pronounced for low stiffness materials than for high stiffness ones. Further, for the same stiffness, the influence of strain-rate-sensitive parameters on the number of loading cycles is more pronounced for small maximum applied stress than for large applied stress.

ACKNOWLEDGMENT

This work was performed under the auspices of the National Science Council, Taiwan, Republic of China.

REFERENCES

1. S.S. Wang and E.S-M. Chim, J. Comp. Mater. **17,** 114 (1983).
2. R. Talreja, *Fatigue of Composite Materials* (Technomic Publishing, Lancaster, PA, 1987).
3. H.T. Hahn, *ASTM STP 674* (American Society for Testing and Materials, Philadelphia, PA, 1979), p. 387.
4. W.W. Stinchcomb and K.L. Reifsnider, *ASTM STP 675* (American Society for Testing and Materials, Philadelphia, PA, 1979), p. 762.
5. K.L. Reifsnider, E.G. Henneke, W.W. Stinchcomb, and J.C. Duke, *Mechanics of Composite Materials* (Pergamon Press, New York, 1983), p. 399.
6. A. Rotem, in *Mechanics of Composite Materials, Recent Advances,* edited by Z. Hashin and C. T. Herakovich (Pergamon Press, New York, 1983), p. 421.
7. S.L. Ogin, P.A. Smith, and P.W.R. Beaumont, Comp. Sci. Technol. **22,** 23 (1985).
8. L. Ye, Comp. Sci. Technol. **36,** 339 (1989).
9. P.T. Curtis, Int. J. Fatigue. **13,** 377 (1991).
10. X.X. Diao, L. Ye, and Y.W. Mai, Compos. Sci. Technol. **54,** 329 (1995).
11. M.P. Wnuk, J. Appl. Mech. **41,** 234 (1974).
12. A. Lowe, O.H. Kwon, and Y.W. Mai, Polymer **37,** 565 (1996).
13. I.M. Daniel and A. Charewicz, Eng. Fracture Mech. **25,** 793 (1986).
14. M.A. Miner, J. Appl. Mech. **12,** 159 (1945).